Kok Wai Wong
B. Sumudu U. Mendis
Abdesselam Bouzerdoum (Eds.)

LNCS 6444

Neural Information Processing

Models and Applications

17th International Conference, ICONIP 2010
Sydney, Australia, November 2010
Proceedings, Part II

2 Part II

Springer

Investment Appraisal under Uncertainty – A Fuzzy Real Options Approach

Shu-Hsien Liao and Shiu-Hwei Ho

Graduate Institute of Management Sciences, Tamkang University
No.151, Ying-chuan Road Tamsui, Taipei County 25137, Taiwan
Michael@mail.tku.edu.tw, succ04.dba@msa.hinet.net

Abstract. The main purpose of this paper is to propose a fuzzy approach for investment project valuation in uncertain environments from the aspect of real options. The traditional approaches to project valuation are based on discounted cash flows (DCF) analysis which provides measures like net present value (NPV) and internal rate of return (IRR). However, DCF-based approaches exhibit two major pitfalls. One is that DCF parameters such as cash flows cannot be estimated precisely in the uncertain decision making environments. The other one is that the values of managerial flexibilities in investment projects cannot be exactly revealed through DCF analysis. Both of them would entail improper results on strategic investment projects valuation. Therefore, this paper proposes a fuzzy binomial approach that can be used in project valuation under uncertainty. The proposed approach also reveals the value of flexibilities embedded in the project. Furthermore, this paper provides a method to compute the mean value of a project's fuzzy expanded NPV that represents the entire value of project. Finally, we use the approach to practically evaluate a project.

Keywords: Project valuation, Real options, Fuzzy numbers, Flexibility, Uncertainty.

1 Introduction

DCF-based approaches to project valuation implicitly assume that a project will be undertaken immediately and operated continuously until the end of its expected useful life, even though the future is uncertain. By treating projects as independent investment opportunities, decisions are made to accept projects with positive computed NPVs. Traditional NPV techniques only focus on current predictable cash flows and ignore future managerial flexibilities, therefore, may undervalue the projects and mislead the decision makers.

Since DCF-based approaches ignore the upside potentials of added value that could be brought to projects through managerial flexibilities and innovations, they usually underestimate the upside value of projects [1, 2]. In particular, as market conditions change in the future, investment project may include flexibilities by which project value can be raised. Such flexibilities are called real options or strategic options. The real options approach to projects valuation seeks to correct the deficiencies of the

K.W. Wong et al. (Eds.): ICONIP 2010, Part II, LNCS 6444, pp. 716–726, 2010.

traditional valuation methods through recognizing that managerial flexibilities can bring significant values to projects.

In DCF, parameters such as cash flows and discount rates are difficult to estimate [3]. These parameters are essentially estimated under uncertainty. With respect to uncertainty, probability is one way to depict whereas possibility is another. Fuzzy set theory provides a basis for the theory of possibility. By modeling the stock price in each state as a fuzzy number, Muzzioli and Torricelli [4] obtained a possibility distribution of the risk-neutral probability in a multi-period binomial model, then computed the option price with a weighted expected value interval, and thus determined a "most likely" option value within the interval. Muzzioli and Reynaerts [5] also addressed that the key input of the multi-period binomial model is the volatility of the underlying asset, but it is an unobservable parameter. Providing a precise volatility estimate is difficult; therefore, they used a possibility distribution to model volatility uncertainty and to price an American option in a multi-period binomial model. Carlsson and Fuller [3] mentioned that the imprecision in judging or estimating future cash flows is not stochastic in nature, and that the use of the probability theory leads to a misleading level of precision. Their study introduced a real option rule in a fuzzy setting in which the present values of expected cash flows and expected costs are estimated by trapezoidal fuzzy numbers. Carlsson et al. [6] also developed a methodology for valuing options on R&D projects, in which future cash flows were estimated by trapezoidal fuzzy numbers.

In addition to the binomial model, the Black-Scholes model [7] is another way to evaluate the option's value. Wu [8] applied the fuzzy set theory to the Black-Scholes formula. Lee et al. [9] adopted the fuzzy decision theory and Bayes' rule as a basis for measuring fuzziness in the practice of option analysis. The Black-Scholes models are used to evaluate simple real option scenarios such as delay decisions, research and development, licenses, patents, growth opportunities, and abandonment scenarios [10]. Despite its theoretical appeal, however, the practical use of real option valuation techniques in industry has been limited by the complexity of these techniques, the resulting lack of intuition associated with the solution process, or the restrictive assumptions required for obtaining analytical solutions. On the other hand, Cox et al. [11] developed a binomial discrete-time option valuation technique that has gained similar popularity to evaluate real options due to its intuitive nature, ease of implementation, and wide applicability to variety of option attributes. In addition, analytical models such as the Black-Scholes formula focus on a single option and cannot deal with multi-option situations.

2 The Valuation Approach

In considering option value, the traditional NPV can be expanded as: expanded NPV = static NPV + value of option from active management [1]. The expanded NPV is also called strategic NPV. Static NPV is the NPV obtained using the traditional discount method; it is also called passive NPV. In this study, a fuzzy binomial valuation approach is proposed to evaluate investment projects that are embedded with real options. The value of the project is represented by its expanded NPV, which can be evaluated by the valuation approach. However, the parameters are estimated by fuzzy numbers when the expanded NPV is estimated; thus, the expanded NPV is called fuzzy expanded NPV (*FENPV*) in this study.

The proposed valuation approach is based on Cox et al. [11]. Assuming there is a call option with the present value of underlying asset S_0 and exercising price K, the value of the underlying asset has P_u probability to rise to uS_0 or P_d probability to drop to dS_0 in the next period. The factors u and d represent the jumping up and down factors of the underlying asset's present value, respectively. The option will be exercised at period t = 1 if the underlying value is higher than K, and forgone if the underlying value is lower than K. The dynamics of the option value is shown in Fig. 1.

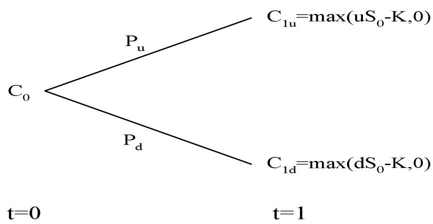

Fig. 1. The dynamics of option value

If the option is sold at price C_0, then the pricing approach is generally based on the assumption of replicating portfolio and can thus be determined by the following expression

$$C_0 = \frac{1}{(1+r)}\left[P_u C_{1u} + P_d C_{1d}\right] \tag{1}$$

in which r is risk-free interest rate, and P_u and P_d are risk-neutral probabilities, which are determined by the following formulas.

$$P_u = \frac{(1+r)-d}{(u-d)} \tag{2}$$

$$P_d = \frac{u-(1+r)}{(u-d)} = 1 - P_u \tag{3}$$

Therefore, the price or present value of the call option is the discounted result of the option values C_{1u} and C_{1d} with risk-neutral probabilities. Also, under the assumption of no arbitrage opportunities, the condition $0<d<1<(1+r)<u$ must be satisfied. Furthermore, the expected return of the underlying asset should be zero based on the no-arbitrage assumption:

$$P_u\left(\frac{uS_0}{1+r} - S_0\right) + P_d\left(\frac{dS_0}{1+r} - S_0\right) = 0 \tag{4}$$

That is

$$\frac{uP_u}{1+r} + \frac{dP_d}{1+r} = 1 \tag{5}$$

Thus, we have the following risk-neutral probabilities equations:

$$\begin{cases} P_u + P_d = 1 \\ \dfrac{uP_u}{1+r} + \dfrac{dP_d}{1+r} = 1 \end{cases} \tag{6}$$

From (1), (2) and (3), we know that the main factors affecting the call option value are jumping factors u and d; it is not easy, however, to estimate their values in a precise manner due to the uncertainty of the underlying volatility.

The cash flow models applied to many financial decision making problems often involve some degree of uncertainty. In the case of deficient data, most decision makers tend to rely on experts' knowledge of financial information when carrying out their financial modeling activities. The nature of this knowledge often tends to be vague rather than random. Hence, this study considers possibilistic uncertainty rather than probabilistic uncertainty and employs fuzzy numbers instead of statistics to estimate the parameters. For lightening computation efforts, we utilize the triangular fuzzy numbers $\tilde{u} = [u_1, u_2, u_3]$ and $\tilde{d} = [d_1, d_2, d_3]$ to represent the jumping factors of the underlying asset. Therefore, the risk-neutral probabilities equations can be rewritten as

$$\begin{cases} \tilde{P}_u \oplus \tilde{P}_d = \tilde{1} \\ \dfrac{\tilde{u} \otimes \tilde{P}_u}{1+\mathrm{r}} \oplus \dfrac{\tilde{d} \otimes \tilde{P}_d}{1+\mathrm{r}} = \tilde{1} \end{cases} \tag{7}$$

where $\tilde{P}_u = [P_{u1}, P_{u2}, P_{u3}]$ and $\tilde{P}_d = [P_{d1}, P_{d2}, P_{d3}]$. Thus, we have

$$\begin{cases} [P_{u1}, P_{u2}, P_{u3}] \oplus [P_{d1}, P_{d2}, P_{d3}] = [1,1,1] \\ \dfrac{[u_1, u_2, u_3] \otimes [P_{u1}, P_{u2}, P_{u3}]_u}{1+\mathrm{r}} \oplus \dfrac{[d_1, d_2, d_3] \otimes [P_{d1}, P_{d2}, P_{d3}]}{1+\mathrm{r}} = [1,1,1] \end{cases} \tag{8}$$

which are

$$\begin{cases} P_{ui} + P_{di} = 1 \\ \dfrac{u_i \times P_{ui}}{1+\mathrm{r}} + \dfrac{d_i \times P_{di}}{1+\mathrm{r}} = 1 \end{cases} \quad \text{for } i = 1,2,3 \tag{9}$$

It can be solved by considering the following relationship.

$$P_{ui} = \frac{(1+\mathrm{r}) - d_i}{u_i - d_i} \tag{10}$$

$$P_{di} = \frac{u_i - (1+\mathrm{r})}{u_i - d_i} \tag{11}$$

Since the risk-free interest rate r and the exercising price K are usually known, they are crisp values, whereas, the option values C_{1u} and C_{1d} become fuzzy numbers as a result of the jumping factors being fuzzified. That is, $\tilde{C}_{1u} = \max(\tilde{u}\mathrm{S}_0 - \mathrm{K}, 0)$ and $\tilde{C}_{1d} = \max(\tilde{d}\mathrm{S}_0 - \mathrm{K}, 0)$. The ranking of two triangular fuzzy numbers $\tilde{A} = [a_1, a_2, a_3]$ and $\tilde{B} = [b_1, b_2, b_3]$ can be derived from $\max(\tilde{A}, \tilde{B}) = [\max(a_1, b_1), \max(a_2, b_2), \max(a_3, b_3)]$. Thus, the pricing formula for the fuzzy call option is

$$\tilde{C}_0 = \frac{1}{1+\mathrm{r}} [\tilde{P}_d \otimes \tilde{C}_{1d} \oplus \tilde{P}_u \otimes \tilde{C}_{1u}] \tag{12}$$

In practical application, the present value of the underlying asset is determined by the NPV of the investment project; the exercising price is the additional outlay to exercise the embedded option.

Managerial flexibility to adopt future actions introduces an asymmetry or skewness in the probability distribution of the project NPV [2]. In the absence of such managerial flexibility, the probability distribution of project NPV would be considerably symmetric. However, in the existence of managerial flexibility such as the exercising of options, enhanced upside potential is introduced and the resulting actual distribution is skewed to the right.

In essence, identical results are obtained in the case of possibilistic distribution which is adopted by this study to characterize the NPV of an investment project. In other words, the characteristic of right-skewed distribution also appears in the *FENPV* of an investment project when the parameters (such as cash flows) are characterized with fuzzy numbers. Although many studies have proposed a variety of methods to compute the mean value [12, 13] and median value [14] of fuzzy numbers, these works did not consider the right-skewed characteristic present in the *FENPV*. Therefore, this study proposes a new method to compute the mean value of the *FENPV* based on its right-skewed characteristic. This mean value can be used to represent the *FENPV* with a crisp value. Moreover, different *FENPV*s can be compared according to their mean values.

Let $\tilde{C} = [c_1(\alpha), c_3(\alpha)]$ be a fuzzy number and $\lambda \in [0, 1]$. Then, the mean value of $\tilde{C}$ is defined as

$$\mathrm{E}(\tilde{C}) = \int_0^1 [(1-\lambda)c_1(\alpha) + \lambda c_3(\alpha)]\mathrm{d}\alpha \tag{13}$$

The weighted index λ is called the pessimistic-optimistic index in [15], but the index is determined by a subjective decision in [15]. However, this study considers that the index can be determined objectively. Fig. 2 illustrates a case in which the *FENPV* is represented by a right-skewed triangular fuzzy number. The right-skewed characteristic of *FENPV*—meaning that the more skew to the right, the more optimistic the payoff of the project—provides a clue to determining λ with $\lambda = \mathrm{A_R} / (\mathrm{A_L} + \mathrm{A_R})$, where $\mathrm{A_L}$ and $\mathrm{A_R}$ are the left-part area and right-part area of the *FENPV*, respectively. Thus, when λ is determined objectively and substituted into (13), the mean value of the *FENPV* can be computed as follows

$$\mathrm{E}(FENPV) = \frac{(1-\lambda)c_1 + c_2 + \lambda c_3}{2} \tag{14}$$

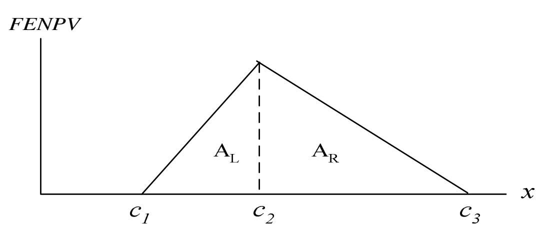

Fig. 2. A *FENPV* with right-skewed distribution

3 Illustrative Examples

An enterprise must continually develop new products and introduce them into the market to create profit. Therefore, evaluating projects of new product development is a crucial task that should be an ongoing effort of an enterprise. In this case, a local biotechnology company in Taiwan proposes a new product development project that needs evaluation. The project must go through two stages before the new product can be introduced into the market. Stage one of the project will require two years and an investment of $I_1 = 40$ (million NT$) toward product development. When this is done, the project will proceed to the second stage, which will require one year and an outlay of $I_2 = 80$ (million NT$) to acquire the equipment and raw material for mass production. Experts estimate that the project will create cash inflows with a present value of 100 (million NT$). If we use the biannual risk-free interest rate r = 3% as the discounting rate and frame six months as one period, the NPV of the project can be calculated as follows:

$$\mathrm{NPV} = 100 - 40 - \frac{80}{(1+0.03)^4} = -11.08\ (\text{million}) \tag{15}$$

This negative NPV suggests that the project should be rejected.

The above results are obtained under the assumption that cash inflows can be generated with certainty. However, this assumption is unrealistic. In fact, the cash inflows will vary with fluctuations in market conditions, such as the market demand of the new product. According to experts' estimation, the new product may have a rate of 20% × (1 ± 5%) fluctuation per year with regard to its market demand. Since the volatility is estimated under uncertainty, a triangular fuzzy number is employed to characterize the possibilistic uncertainty of the volatility. Based on the estimation, the triangular fuzzy number $\tilde{\rho} = [(1-0.05)\times 0.2, 0.2, (1+0.05)\times 0.2] = [0.19, 0.2, 0.21]$ is used to express the fuzzy volatility. From the fuzzy volatility $\tilde{\rho}$, the fuzzy jumping factors $\tilde{u}$ and $\tilde{d}$ can be determined as $\tilde{u} = \exp(\tilde{\rho} \otimes \sqrt{\tau})$ and $\tilde{d} = 1/\tilde{u}$, where τ is the chosen time interval expressed in the same unit as $\tilde{\rho}$ and exp denotes the exponential function. In this case, the value of τ is 0.5 because there are six months (0.5 year) in each period. As a result, we have $\tilde{u} = [1.1438, 1.1519, 1.1601]$ and $\tilde{d} = [0.8620, 0.8681, 0.8743]$. The fuzzy risk-neutral probabilities are $\tilde{P}_u = [0.5448, 0.5704, 0.5962]$ and $\tilde{P}_d = [0.4038, 0.4296, 0.4552]$, respectively. With the above conditions, a binomial tree of the project's cash inflows can be established, as shown in Fig. 3. (For simplicity, the numbers in the binomial tree are represented to two digits after the decimal point.)

Nevertheless, the project may have some decision flexibilities when the project is undertaken. For instance, when the market conditions are unfavorable, the project can be deferred one period to undertake or the project can abandon its second stage investment to prevent losses from mass production. Therefore, the project with deferring option and abandoning option will be evaluated in the following subsections, respectively. Moreover, the project with a sequential multiple options which is combined with deferring option and abandoning option will also be evaluated.

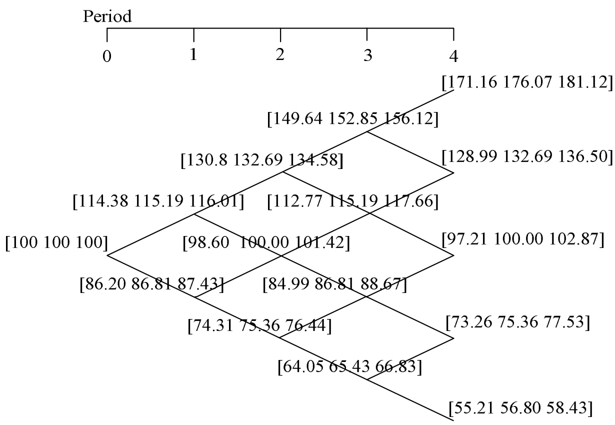

Fig. 3. Binomial tree of the project's cash inflows

3.1 Option to Defer

First of all, considering the situation that decision maker defers one period to undertake the first stage investment and commits to undertake the second stage investment. In this case, the project's total outlay that discounted to period one is calculated as follows:

$$\mathrm{I}_{\_\mathrm{defer}} = 40 \times (1+0.03) + \frac{80}{(1+0.03)^3} = 114.41 \tag{16}$$

The decision tree is shown in Fig. 4, where V=100, $\tilde{I}_{\_defer} = [114.41, 114.41, 114.41]$ and $\tilde{0} = [0, 0, 0]$. The root value in Fig. 4 is the *FENPV* of the project with deferring option and can be calculated as follows:

$$FENPV = [\tilde{P}_u \otimes \tilde{C}_{1u} \oplus \tilde{P}_d \otimes \tilde{C}_{1d}]/(1+0.03) = [0, 0.43, 0.92] \tag{17}$$

The mean value of the *FENPV* is 0.46 (million), and the value of the option to defer the first stage investment is 0.46 – (−11.08) = 11.54 (million NT$).

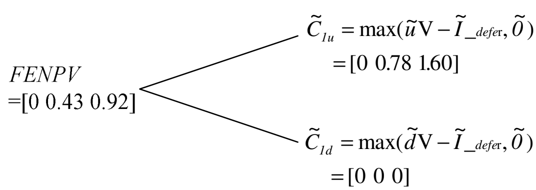

Fig. 4. The decision tree of the project with the option to defer

3.2 Option to Abandon

Furthermore, when the decision maker only possesses the option to abandon the second stage investment, this implies that the decision maker has already completed the first stage investment without deferring.

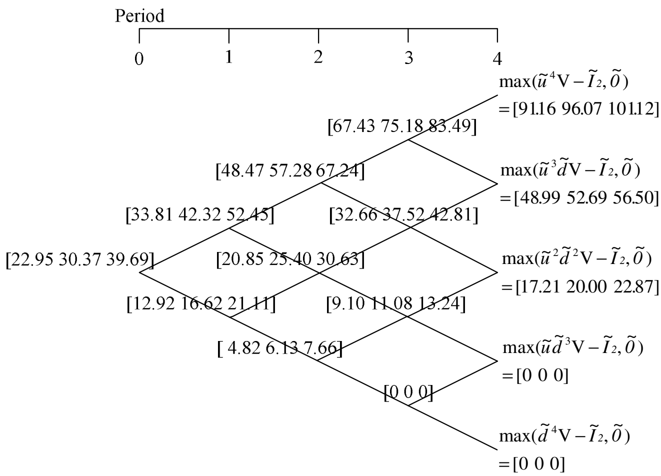

Fig. 5. The decision tree of the project with the option to abandon

The decision tree is shown in Fig. 5, in which $\tilde{I}_2 = [80, 80, 80]$. From the root value in Fig. 5, we can conclude that the *FENPV* of the project with option to abandon the second stage investment is *FENPV* = [22.95, 30.37, 39.69] – $\tilde{I}_1$= [−17.05, −9.64, –0.31], where $\tilde{I}_1 = [40, 40, 40]$. In this case, the mean value of the *FENPV* is −8.68 (million), and thus, the value of the option to abandon the second stage investment is –8.68 – (−11.08) = 2.4 (million).

3.3 Sequential Multiple Options

Finally, when the project involves these two options but with different expiration days, these two options form a sequential multiple options. The decision tree of the sequential multiple options is shown in Fig. 6.

In the sequential multiple options, decision makers have the options not only to abandon the second stage investment but also to defer the first stage investment. Therefore, the decision in period one is $\max(\tilde{C}_{1u} - \tilde{I}_1, \tilde{0})$ and $\max(\tilde{C}_{1d} - \tilde{I}_1, \tilde{0})$, where $\tilde{C}_{1u}$ and $\tilde{C}_{1d}$ are the project values in the up and down cases, respectively, during period one. Based on the values at period two, we can find that $\tilde{C}_{1u} = [33.81, 42.32, 52.45]$ and $\tilde{C}_{1d} = [12.92, 16.62, 21.11]$. The *FENPV* of the project with sequential multiple options is *FENPV* = [0, 1.28, 7.21], its mean value is 3.60 (million), and the value of the sequential multiple options is 3.60 – (−11.08) = 14.68 (million).

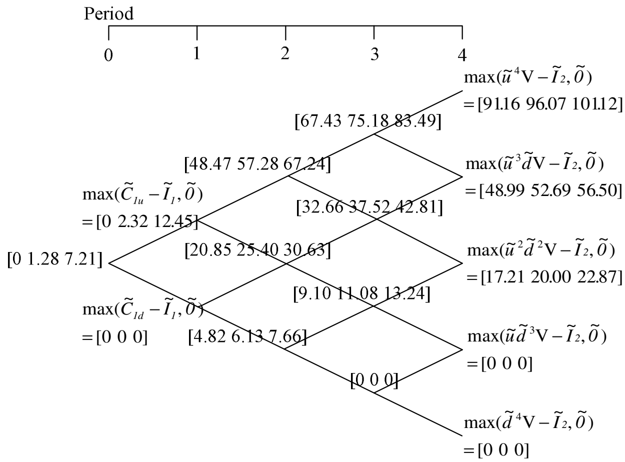

Fig. 6. The decision tree of the project with sequential multiple options

4 Discussion and Conclusion

In Table 1, we summarize the evaluation results of the new product development project that embedded with three different real options, respectively.

Table 1. A summary of the results (in million NT$)

Type of option	*FENPV* of the project	Mean value of the *FENPV*	Option value
Option to defer	[0, 0.43, 0.92]	0.46	11.54
Option to abandon	[−17.05, −9.635, −0.31]	−8.68	2.4
Multiple options	[0, 1.28, 7.21]	3.60	14.68

From the evaluation results, we can observe that if the project does not have any decision flexibility, the project's NPV is −11.08 (million NT$) and the project should therefore be rejected. However, when the project is embedded with some decision flexibilities, the decisions will be different. Confronting uncertain market conditions, the decision flexibilities, such as deferring investment in the first stage or abandoning investment in the second stage, have specific values. In this paper, we have verified the values of these flexibilities from the aspect of fuzzy real options.

When the project involves the option to defer investment in the first stage, the mean value of the project's *FENPV* is 0.46 (million NT$). The overall value of the project is positive, thus, the project become acceptable. Moreover, the value of the option to defer is 11.54 (million NT$). The option value stems from the flexibility that decision maker can defer investment in the first stage to avoid the downward losses at project initiation.

Moreover, when the project includes the option to abandon the second stage investment, the mean value of the project's *FENPV* is −8.68 (million NT$). Although this mean value is negative, it is still greater than the original NPV=−11.08 (million NT$). This reveals that the second stage option can still prevent losses when the market conditions are downward and can retain the upside potential of profit when the market conditions are upward. Therefore, this option to abandon the second stage investment has a value of 2.4 (million NT$)－lower than the value of option to defer. The reason is that the first stage investment has been completed without deferring, no matter what the market conditions are. Thus, even though the market conditions are downward at the initiation of the project, the decision maker will only be able to prevent losses at the second stage. Due to the smaller extent of hedging, the second stage option has a lower option value than the first stage option.

Lastly, when both options form a sequential multiple options, the mean value of the project's *FENPV* is 3.60 (million NT$), which represents the total value of the project. Since this value is positive, the project is acceptable. The value of the sequential multiple options is 14.68 (million NT$). This option value is higher than the value of a single option. This result shows that the multiple options provide greater value than a single option because multiple options provide more flexibility. However, the value of multiple options does not equate directly to the addition of the values of both options. The value cannot be raised linearly because of the nonlinear operations in the valuation model and the trade-off between both options in the hedging process.

In an uncertain economic decision making environment, information such as cash flows, interest rate, cost of capital, and so forth possess some vagueness but not randomness [16]. Consequently, this study has proposed the fuzzy binomial valuation approach to evaluate investment projects with embedded real options in uncertain decision making environments.

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